Estimating Time-Varying Mixed Graphical Models

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Time varying systems of interest in many disciplines!

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Economics

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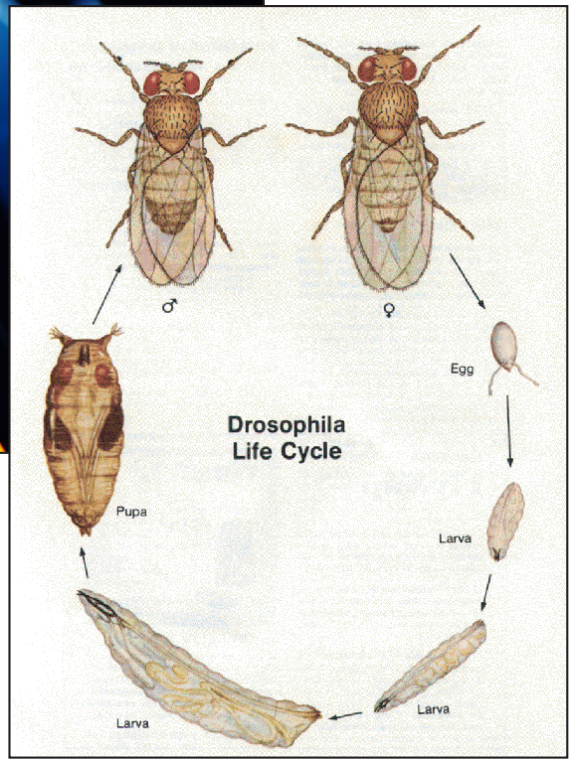
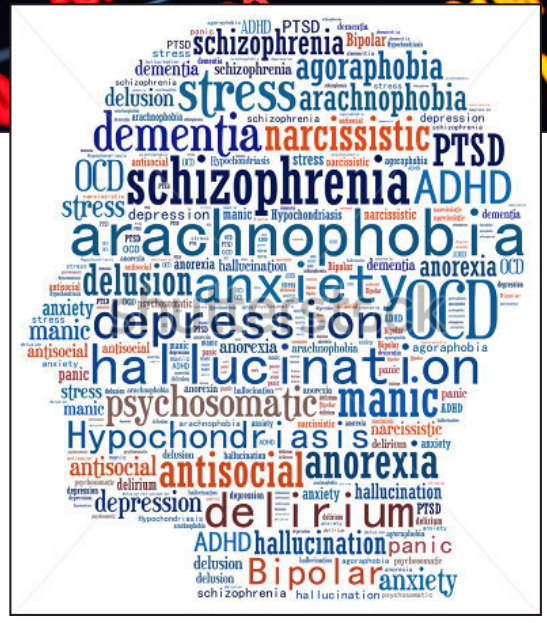
Economics



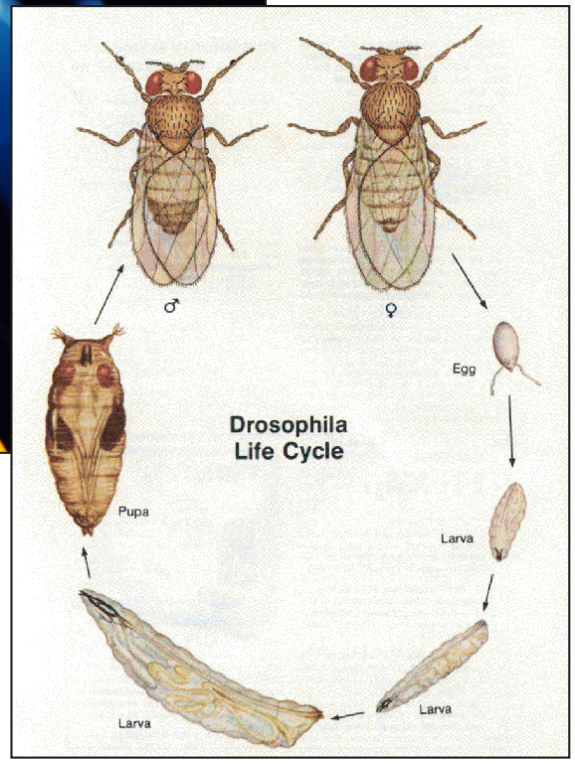
Biology

Time varying systems of interest in many disciplines!

Economics

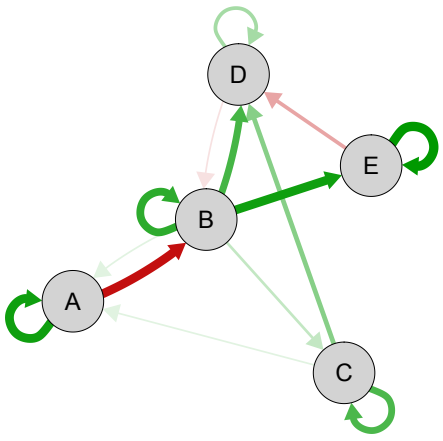


Psychopathology



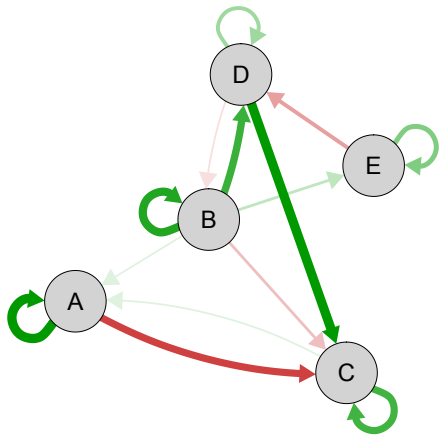
Biology

What is a Time-Varying Model?



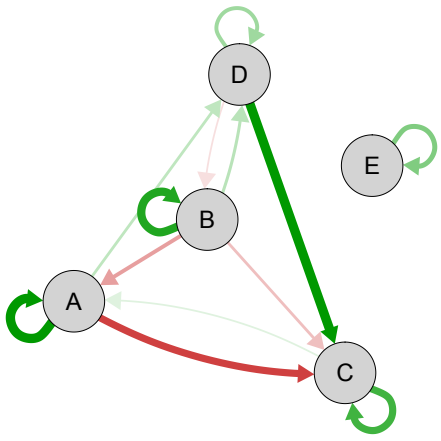
Day 1

...



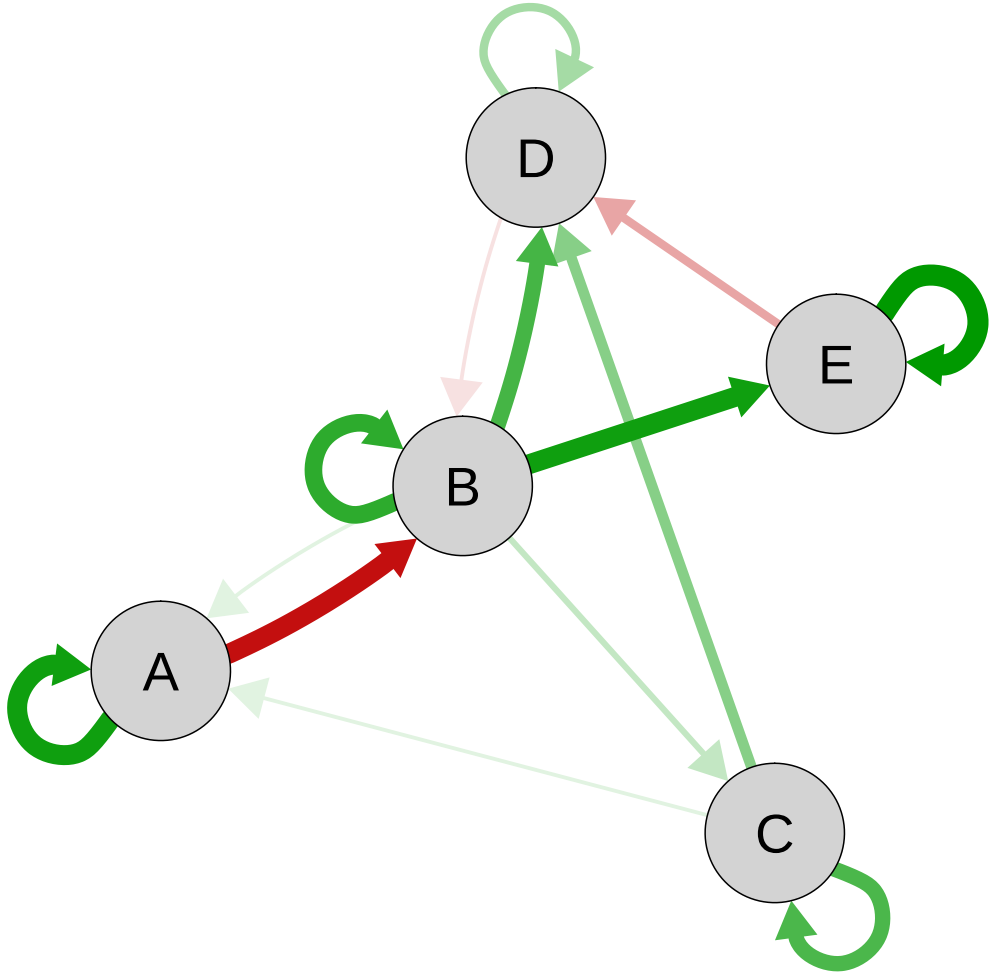
Day 16

...



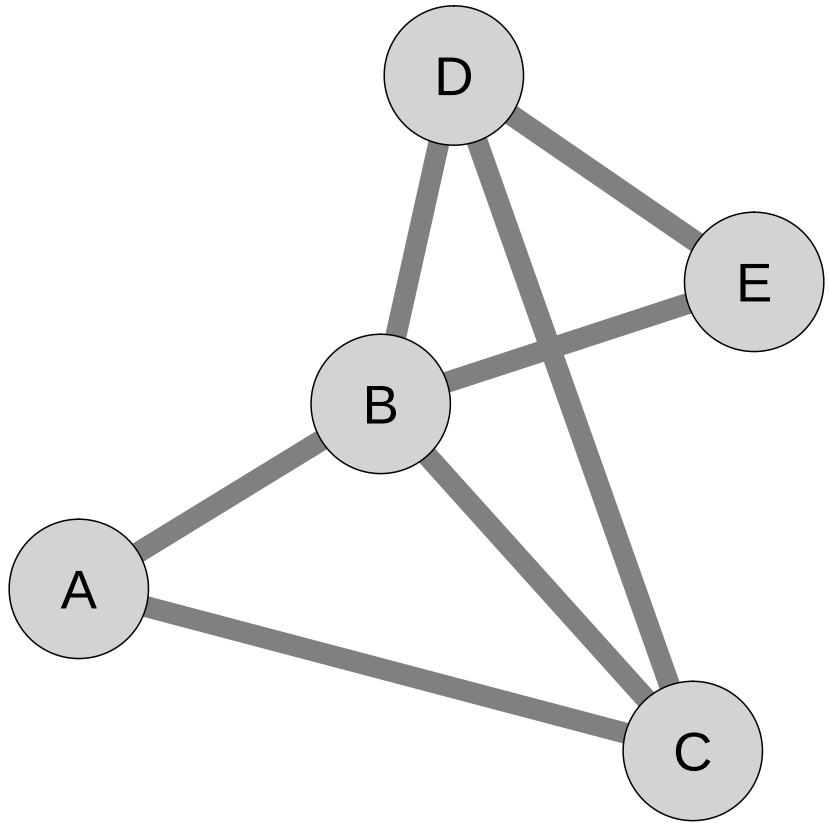
Day 20

But first: Time-**in**variant Model



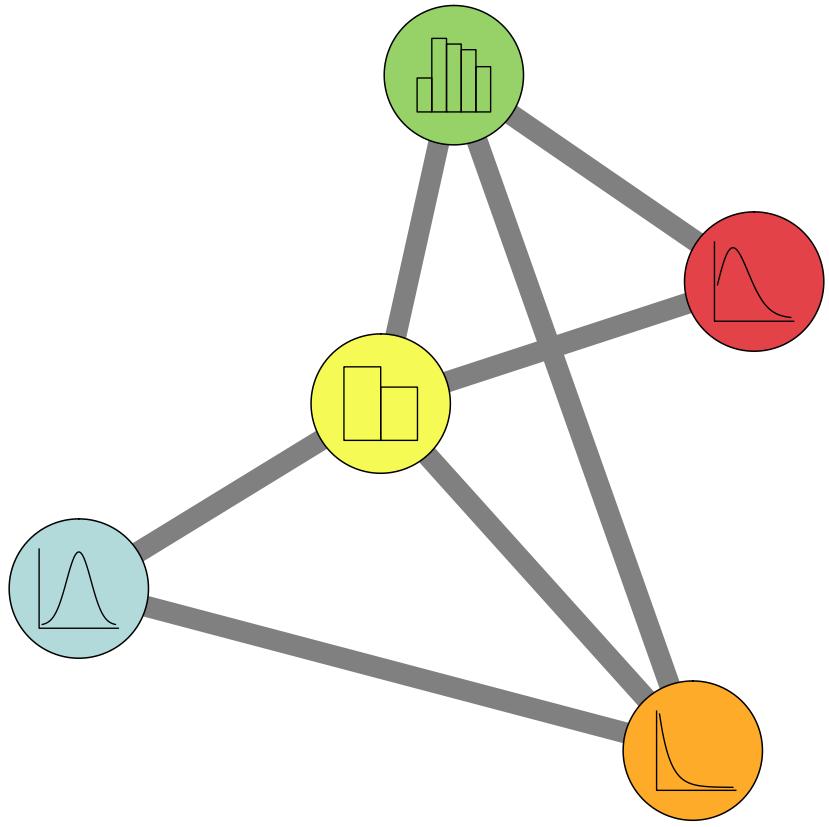
Day 1,2, ..., 19, 20

Approximate True Model with a Graphical Model



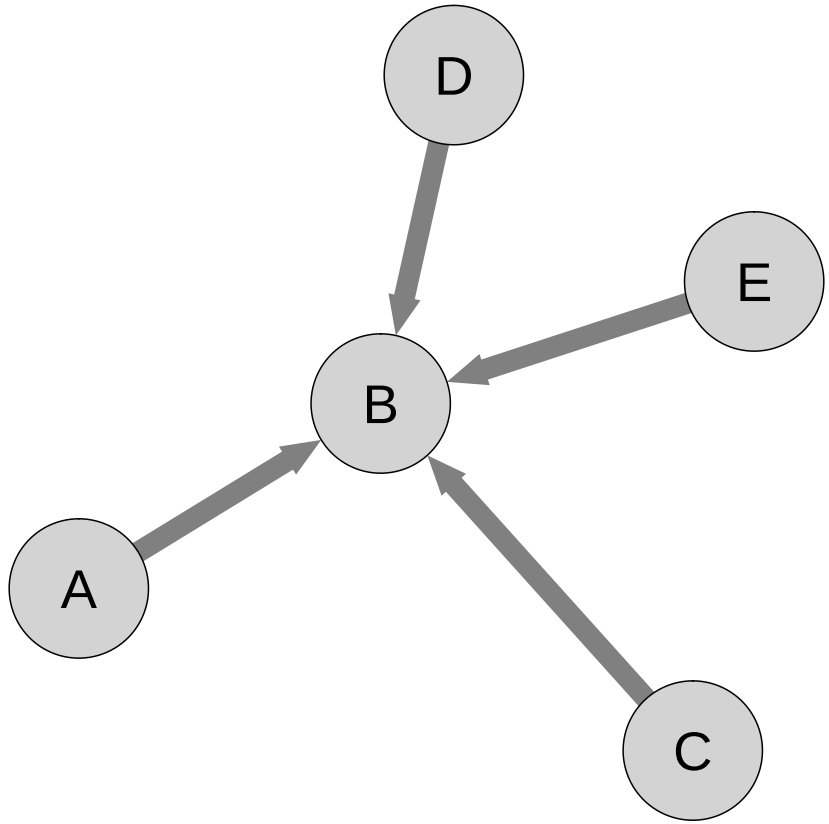
Edge = Conditional Dependence

Mixed Exponential Graphical Model



(Yang et al., 2014)

Neighborhood Regression Method



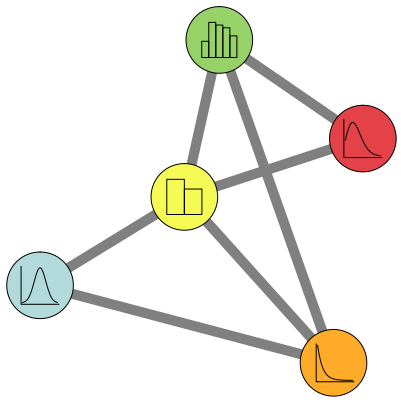
Regress all variables on B

Estimation Algorithm

1. Nodewise regression on all variables:
   - $\min_{(\theta_0,\theta)\in\mathbb{R}^p} \left[\frac{1}{N}\sum_{i=1}^{N}(y_i - \theta_0 - X_{\setminus s;i}^T\theta)^2 + \lambda_n||\theta||_1\right]$
   - Select λ_n using CV or EBIC
2. Threshold Parameter Estimates
   - $\tau_n \asymp \sqrt{d}||\theta||_2\sqrt{\frac{\log p}{n}}$
3. Combine Estimates from both regressions
   - AND-rule: Edge present if both parameters are nonzero
   - OR-rule: Edge present if at least one parameter is nonzero

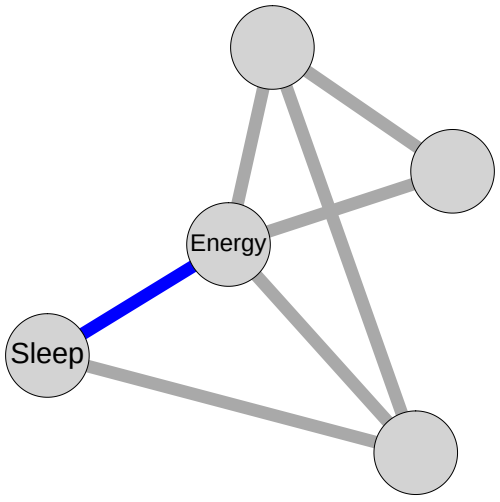
(Loh & Wainwright, 2013; Haslbeck & Waldorp, 2015)

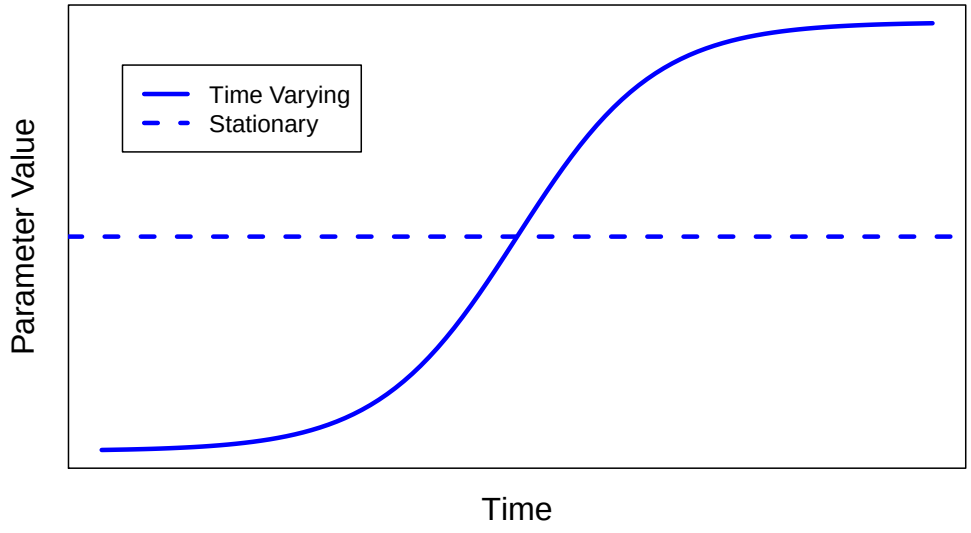
Recab: Time-**in**variant mixed Graphical Model



Time	X_1	X_2	X_3	X_4	X_5
1	3.45	1	0.98	3	1
2	1.11	3	0.82	3	2
⋮	⋮	⋮	⋮	⋮	⋮
$T-1$	0.12	2	0.71	2	2
T	−0.78	1	0.18	1	1

Parameters may change over time!





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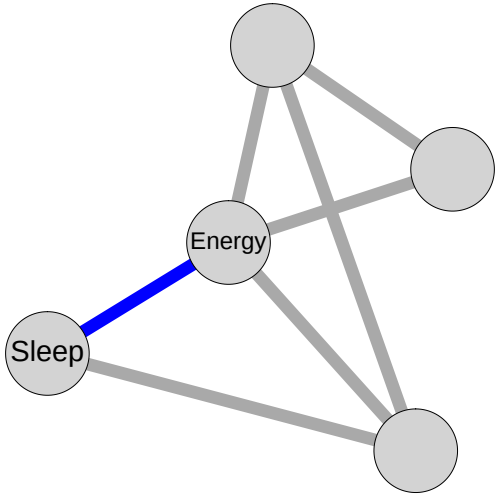

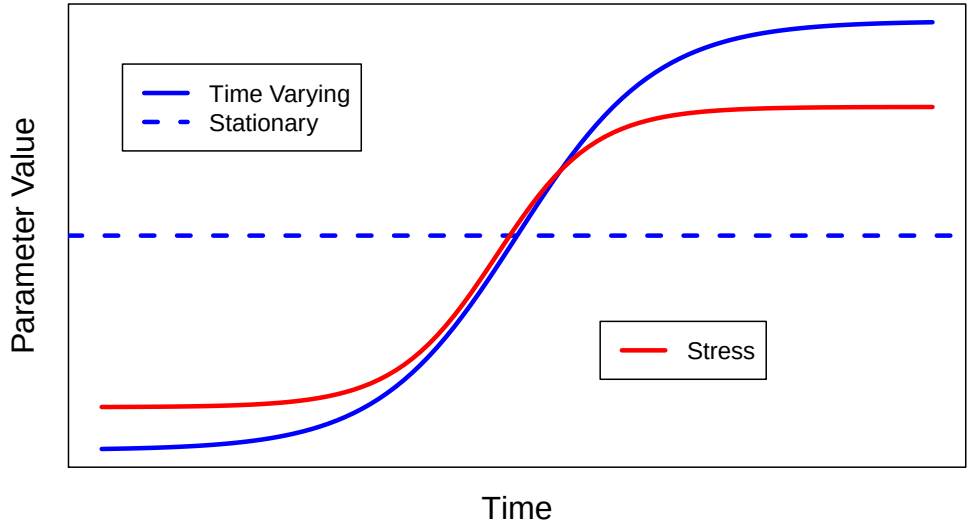

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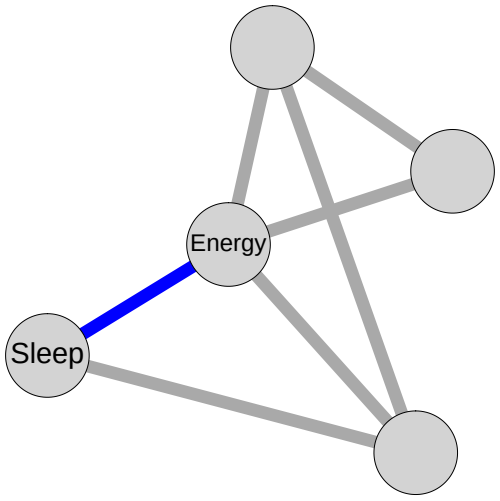
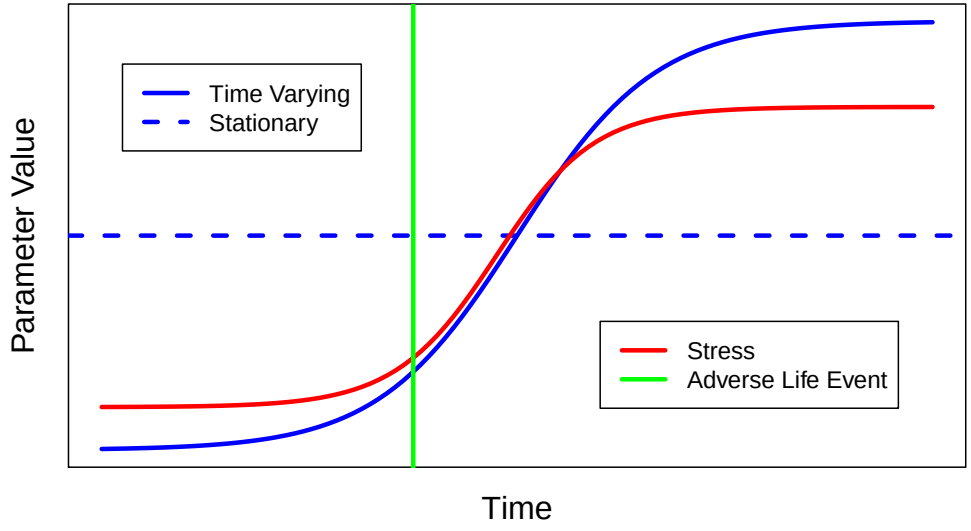
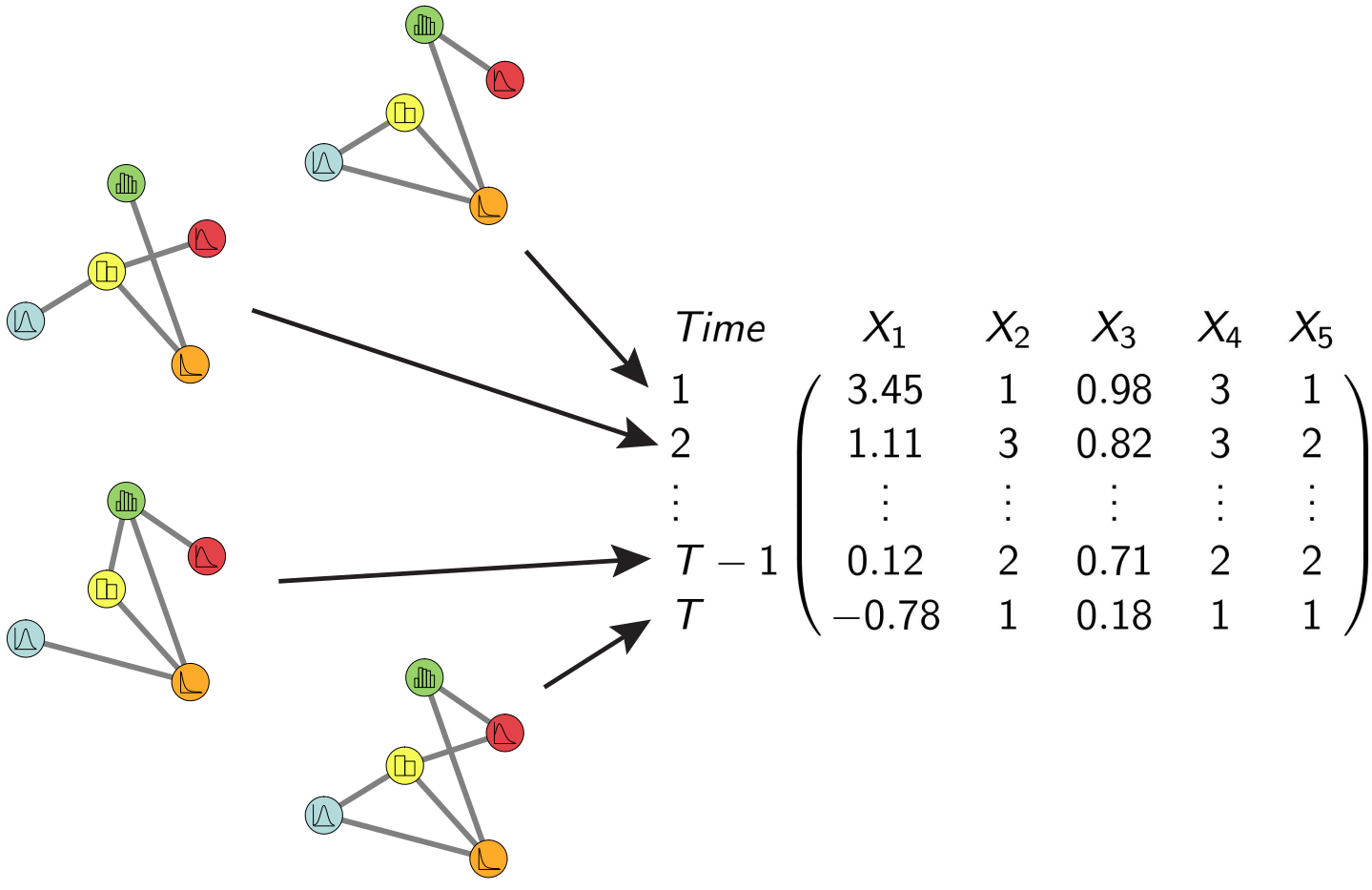




Time-varying mixed Graphical Model



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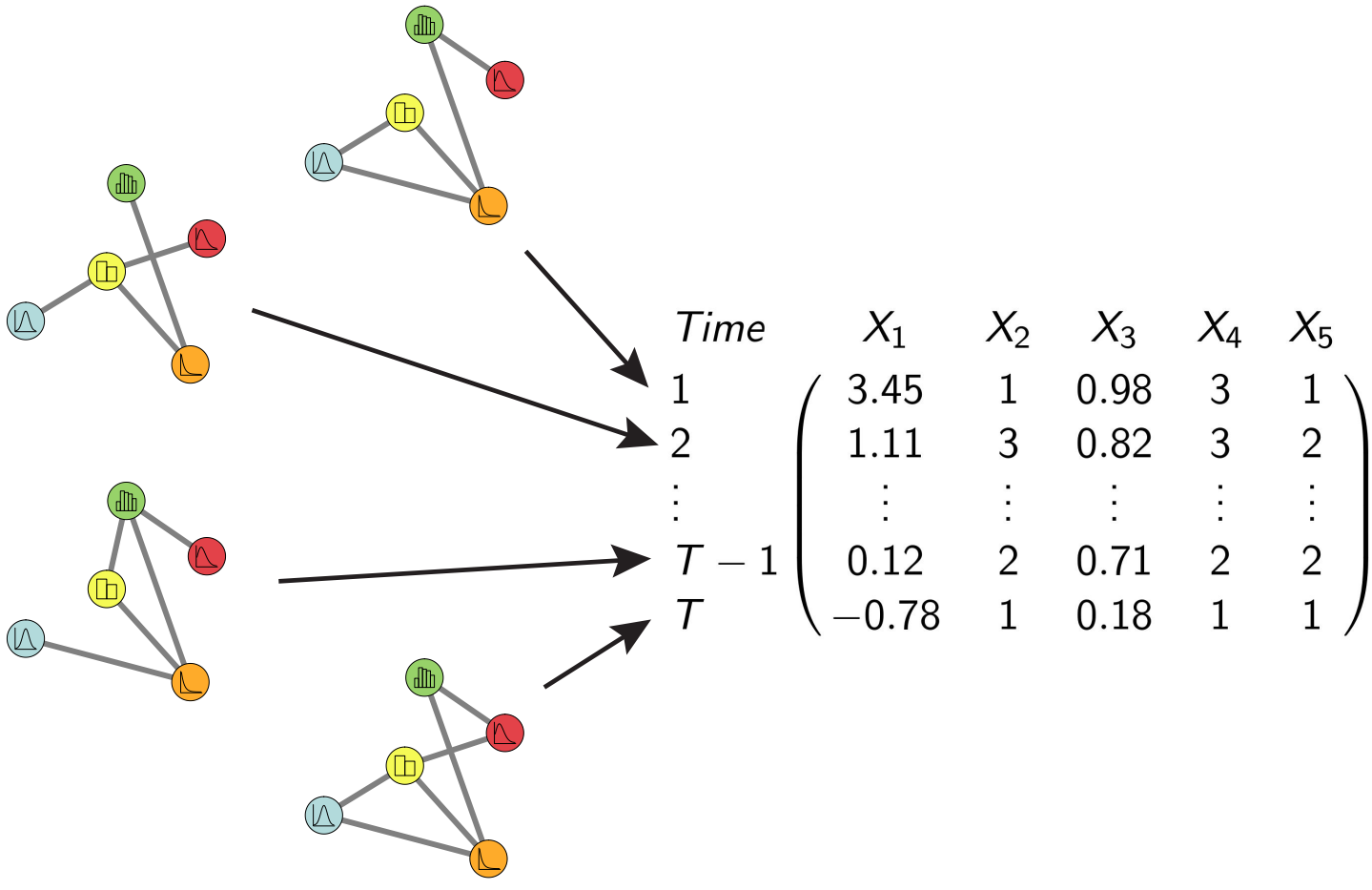

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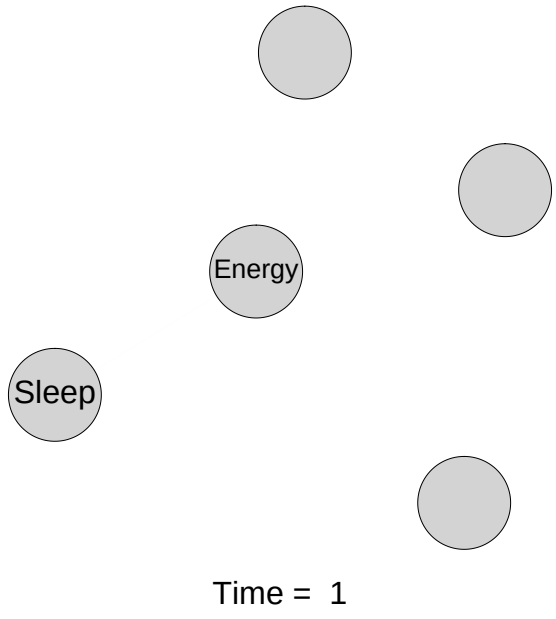
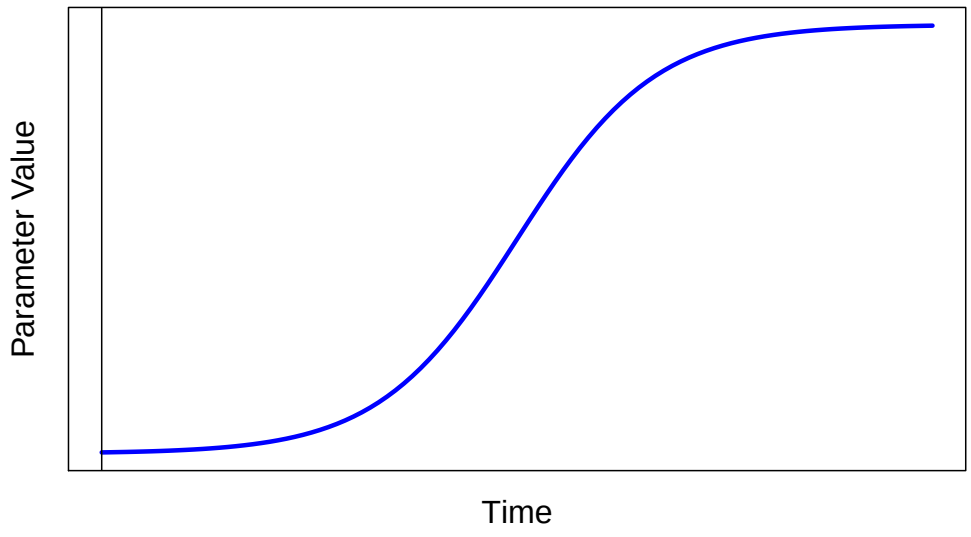
But: we have the scaling $\tau_n \asymp \sqrt{d}||\theta||_2\sqrt{\frac{\log p}{n}}$

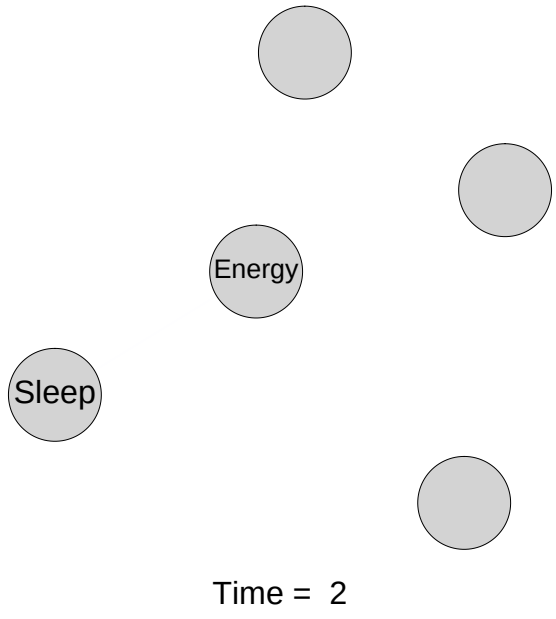
Local Stationarity

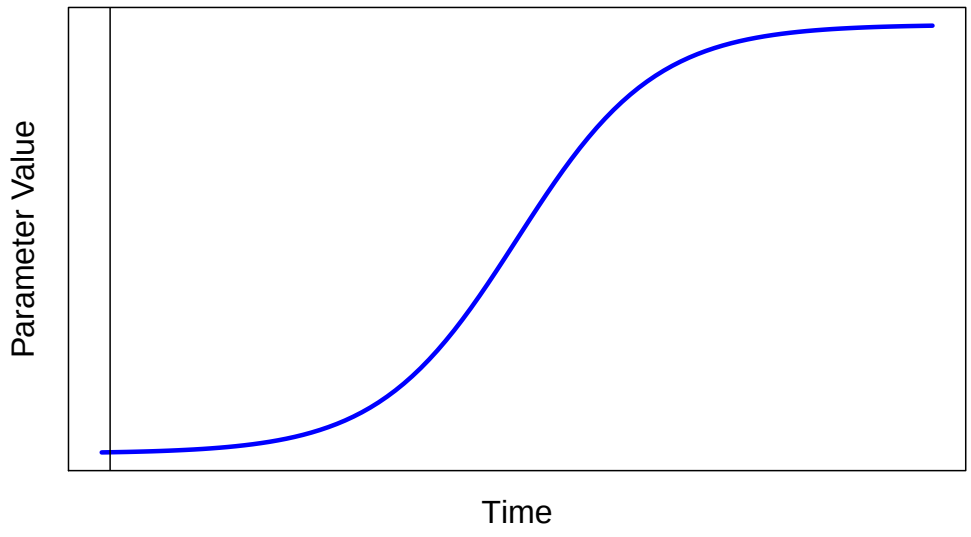




Assumption: Edge parameters are a smooth function of time

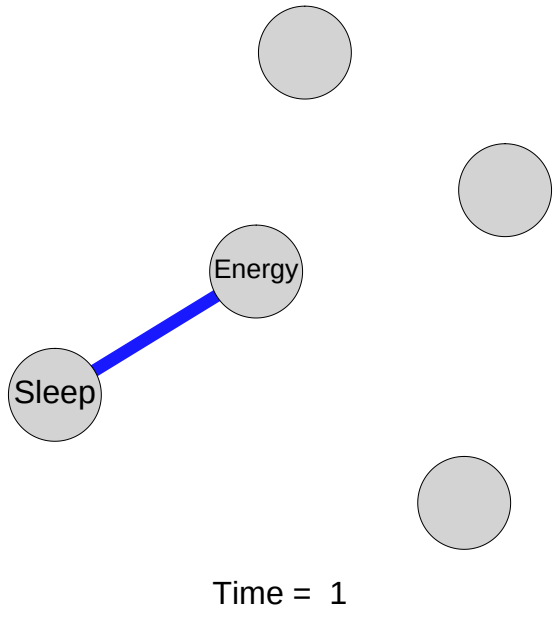
Local Stationarity

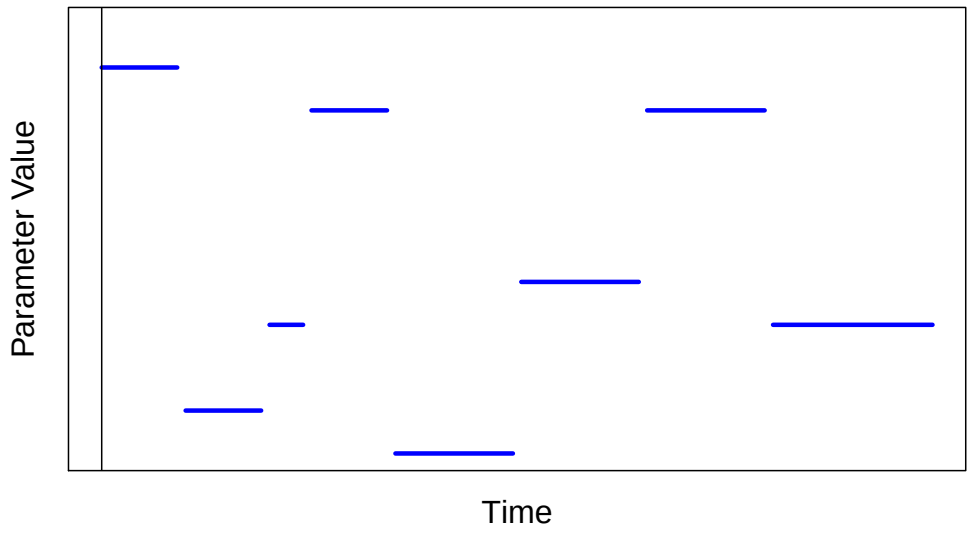




Assumption: Edge parameters are a smooth function of time

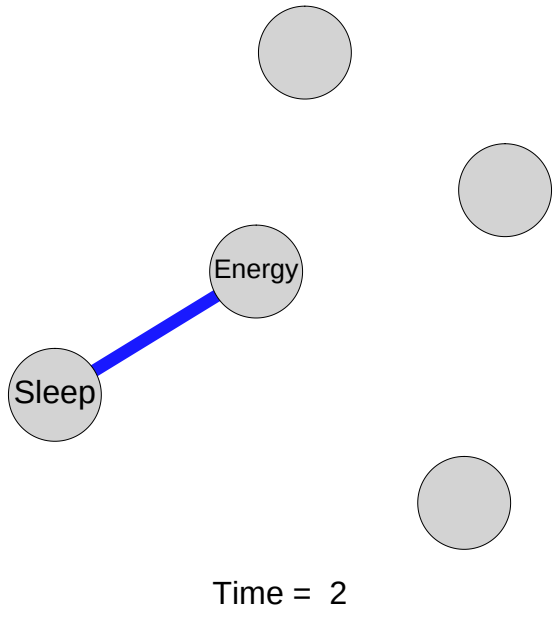
Local Stationarity Violated!

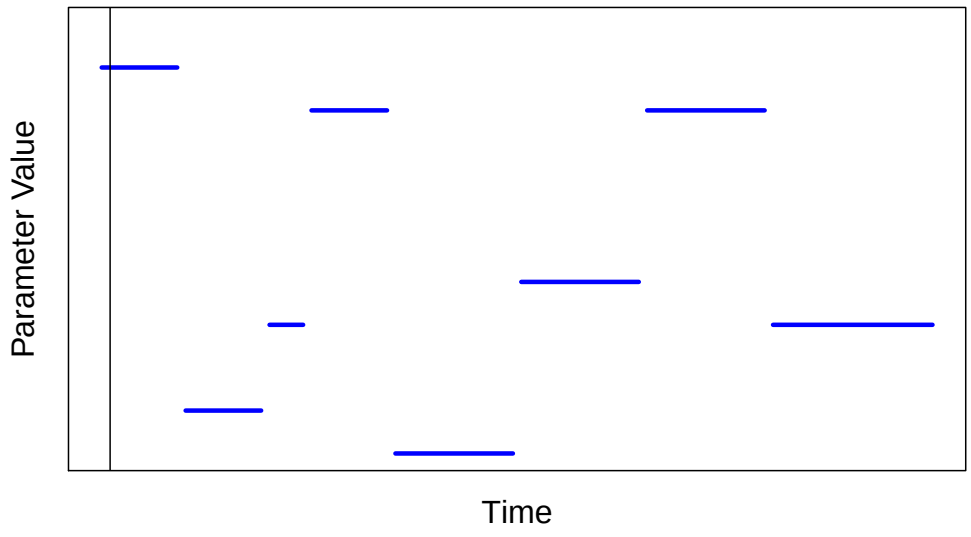




Edge parameter is no smooth function of time!

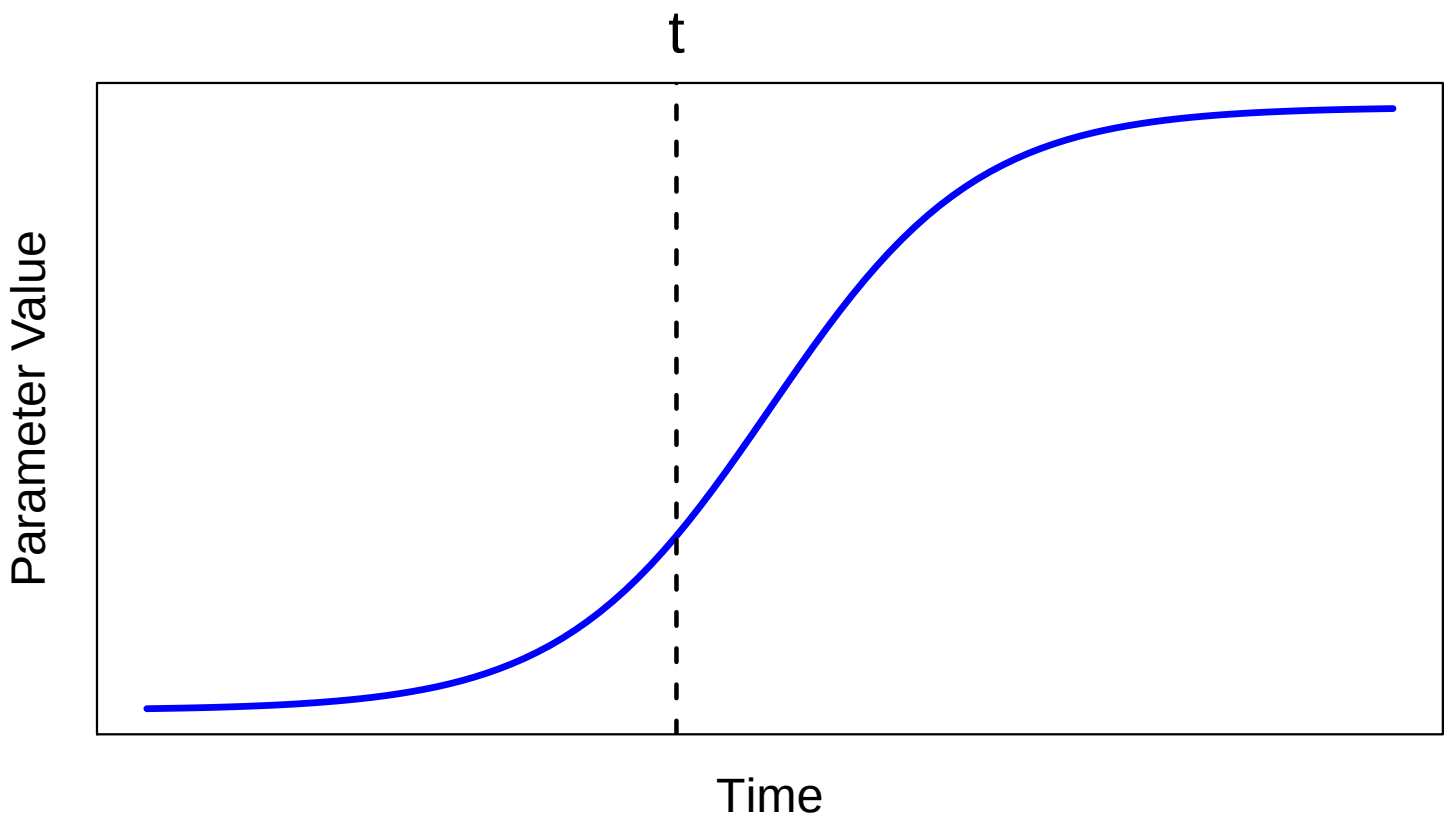
Local Stationarity Violated!



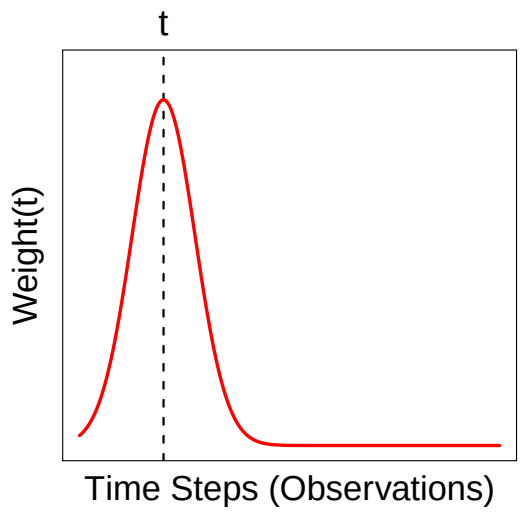


Edge parameter is no smooth function of time!

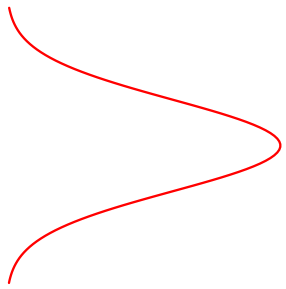
Again: Local Stationarity



Time-varying Graphs via Weighted Regression



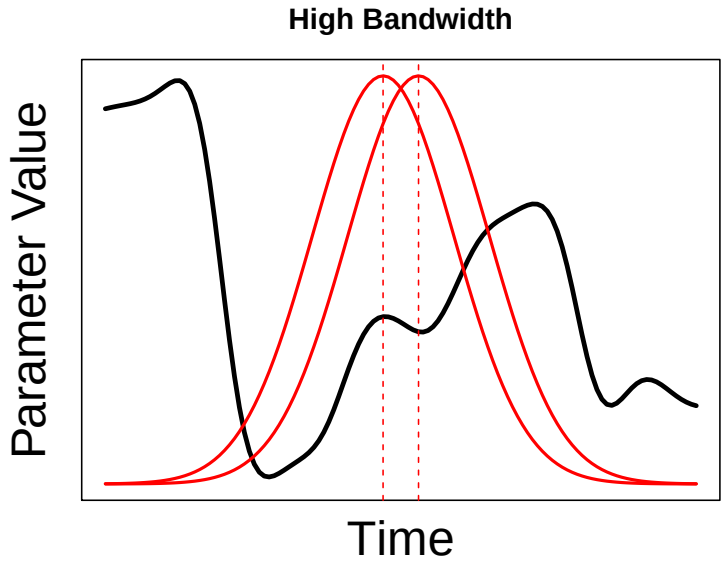
Time	X_1	...	X_5	*Weight*(t)
1	0.92	...	−1.47	0
2	0.78	...	−0.48	0.13
3	0.07	...	0.42	0.60
4	−1.99	...	1.36	1.00
5	0.62	...	−0.10	0.60
6	−0.06	...	0.39	0.13
7	−0.21	...	0.99	0
⋮	⋮	⋮	⋮	⋮
T	−0.16	...	0.18	0

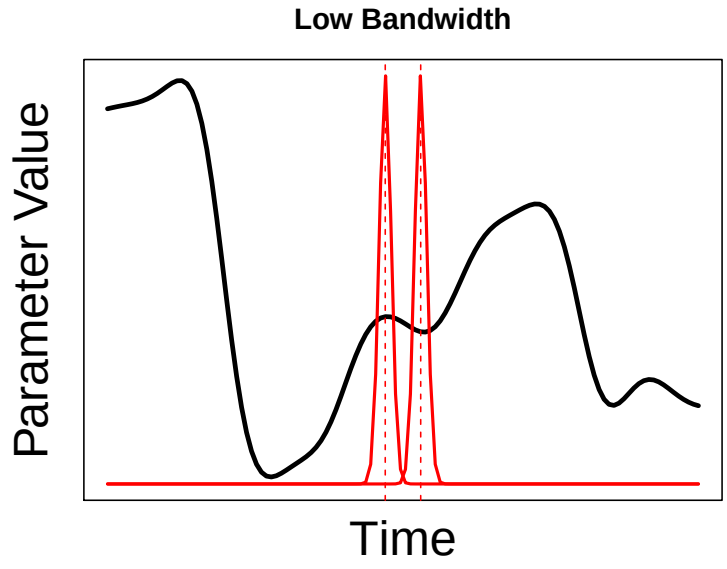


Weighted cost function:

$$\min_{(\theta_0,\theta)\in\mathbb{R}^p} \left[\frac{1}{N} \sum_{i=1}^{N} w_{i;t} (y_{i;t} - \theta_{0;t} - X_{\setminus s;i}^T \theta_t)^2 + \lambda_n ||\theta_t||_1 \right]$$

What is the right bandwidth?



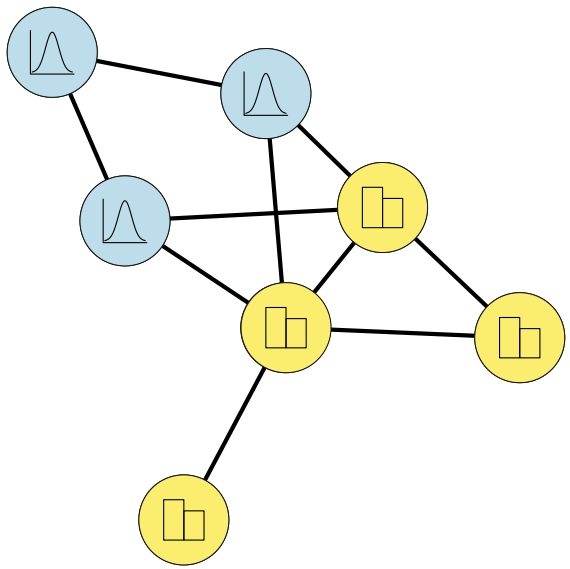


More Information for estimation

Higher sensitivity to change

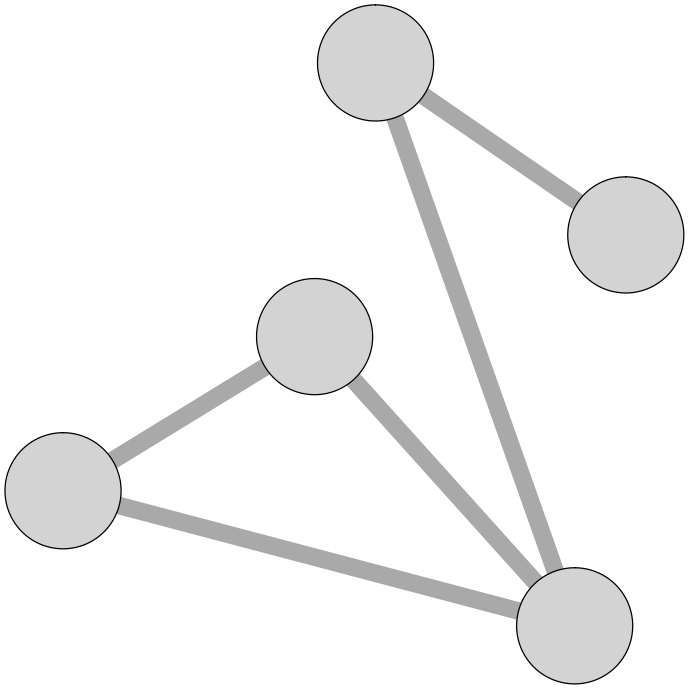
Scaling: $\tau_n \asymp \sqrt{d}||\theta||_2 \sqrt{\frac{\log p}{n}}$

Determine Performance via Simulation

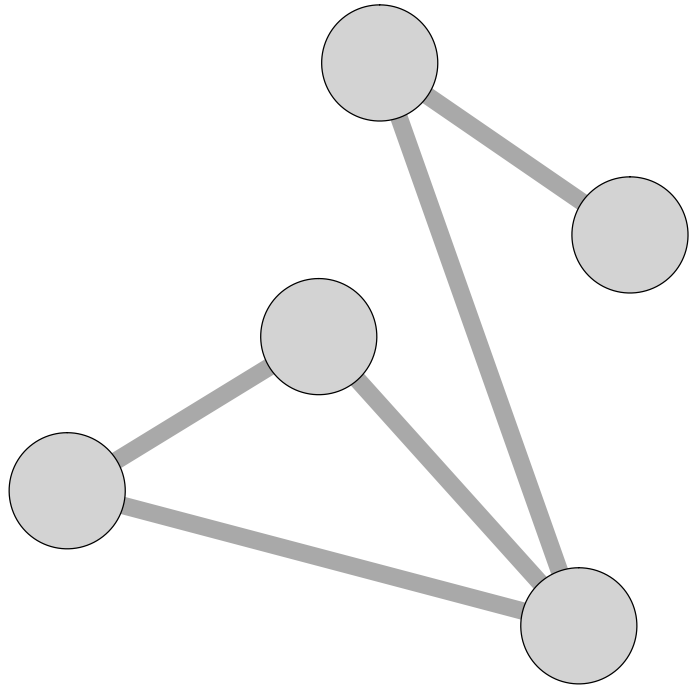


- Binary-Gaussian Graphical Model
- 20 Nodes
- Always 19 edges present
- Of these are always 6 changing
- Type of change: smooth vs. sudden

Simulation: Smooth vs. Sudden Changes



Smooth change

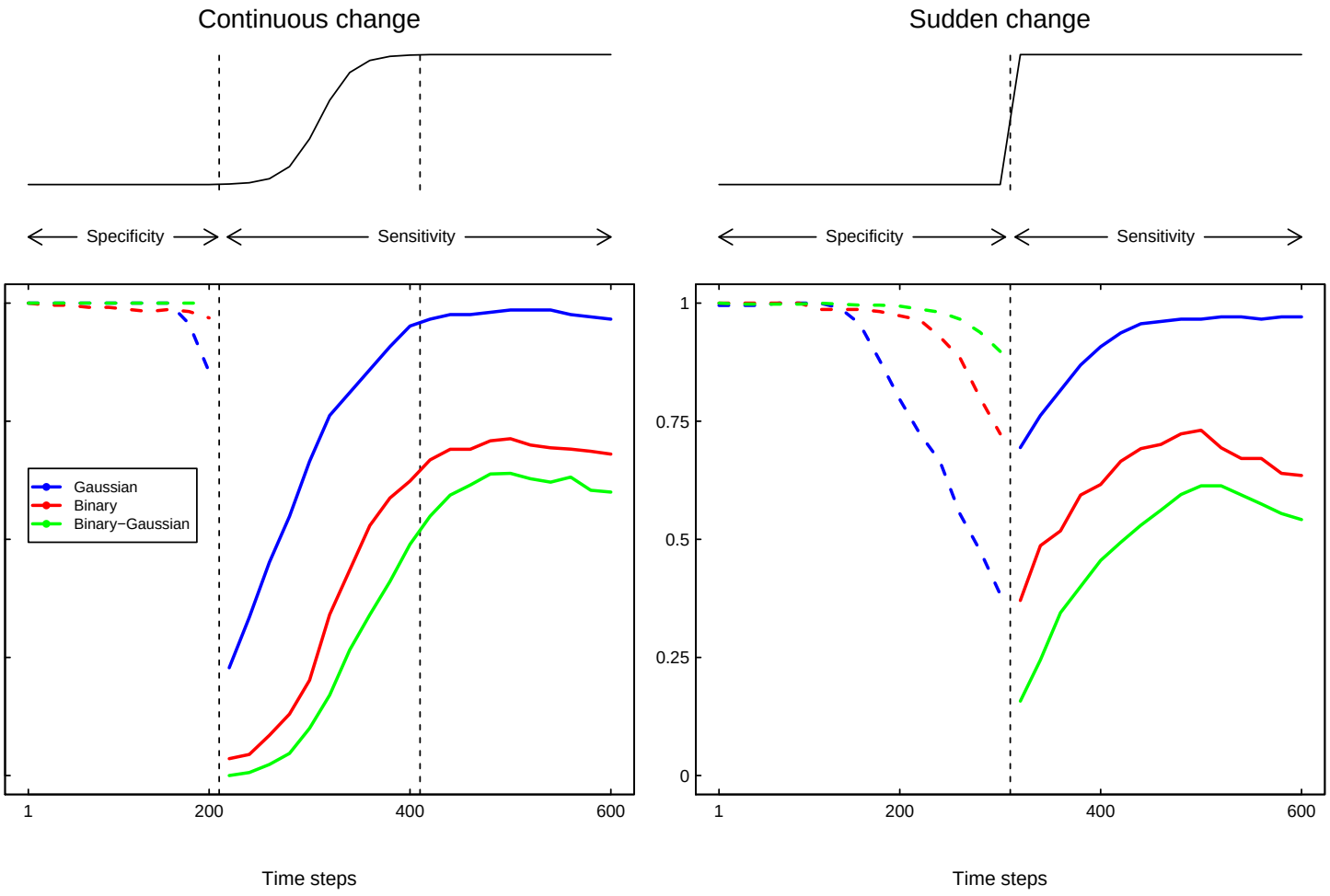


Sudden change

Time = 2

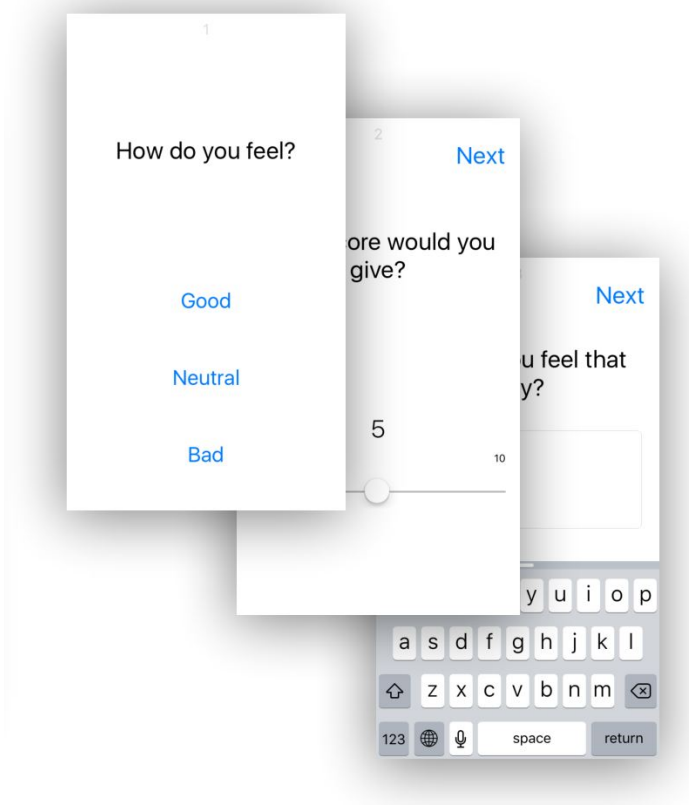
Simulation: Results

N = 600
Nt = 205.9
Nt/p = 10.3



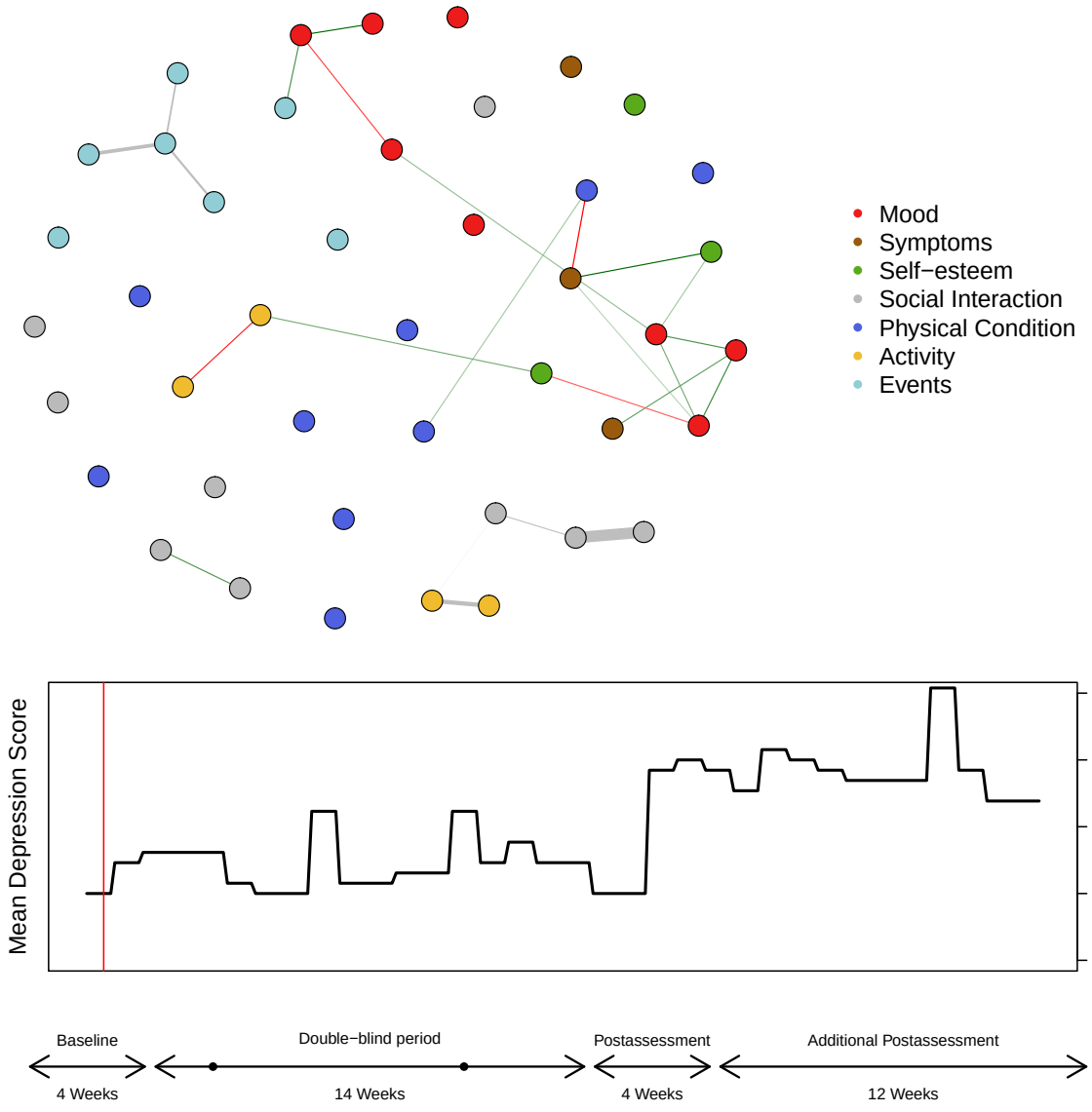
$$\text{bandwidth} = 0.8/N^{1/3}$$

Application to Event Sampling Data



- Time series of 1 person
- 43 variables
- Up to 10 measurements a day
- For 36 weeks

Time-varying Graph of Psychopathology



Time-varying Mixed Graphical Models

Summary

- Estimation of time-varying mixed Graphical Models
- Assumption of local stationarity
- Works in realistic situations

R-package **mgm**

- Available on CRAN: install.packages('mgm')
- Also time-varying VAR models

Contact: jonashaslbeck@gmail.com; http://jmbh.github.io