3
Representations of the Ising Model

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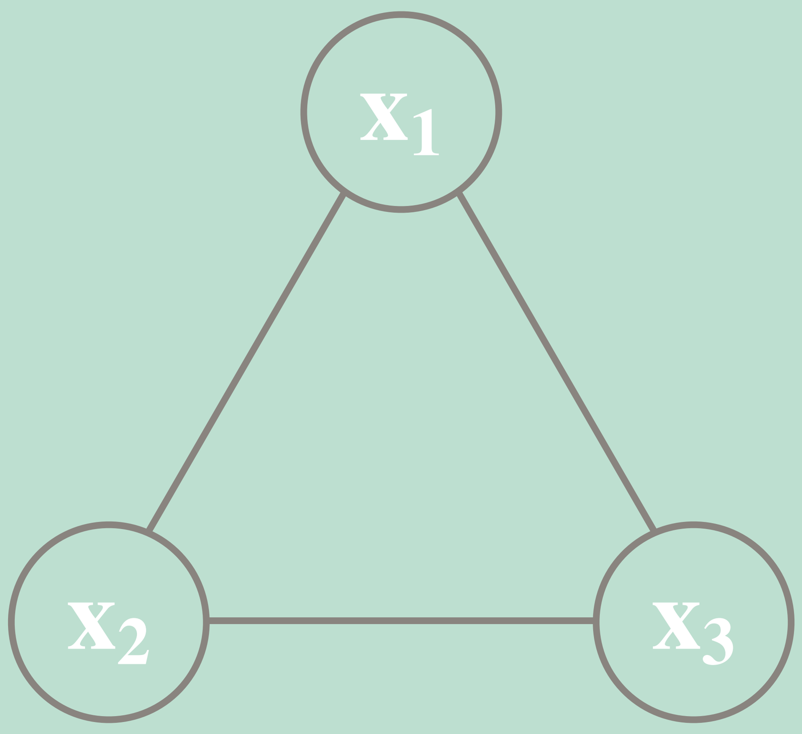
reciprocal causation representation

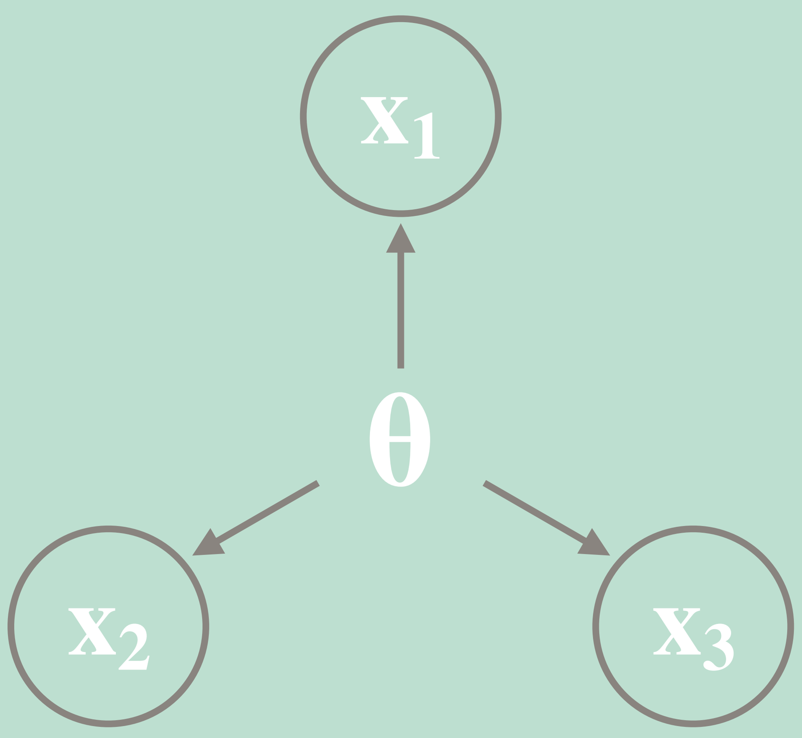
- associations are a consequence of mutualistic relations between the observed variables themselves.

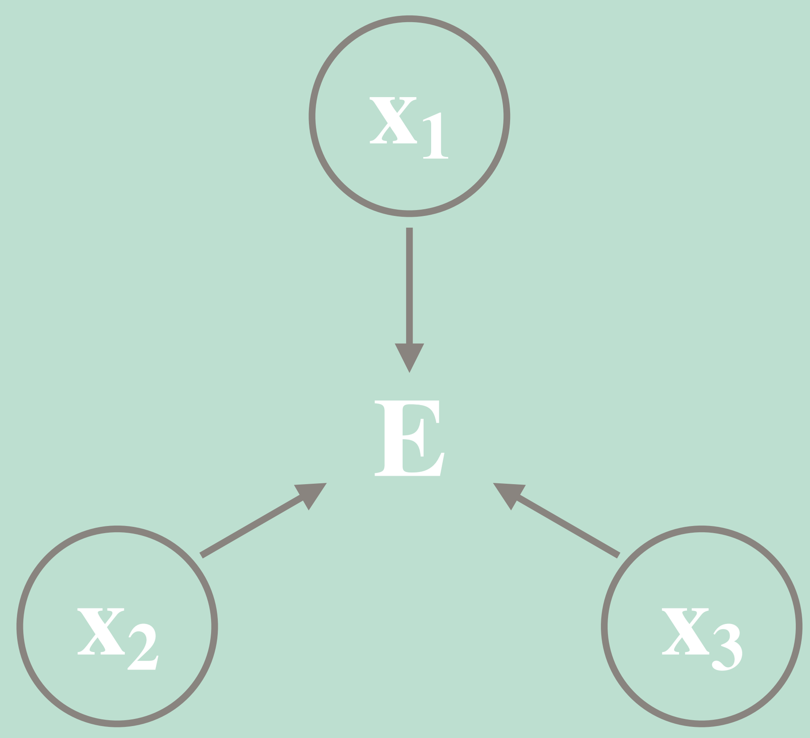
$$p(\mathbf{x}) = \frac{\exp\left(\sum_i x_i b_i + x_+^2\right)}{Z}$$



$$\exp\left(x_+^2\right) = \int_R \frac{\exp\left(2x_+\theta - \theta^2\right)}{\sqrt{\pi}}\, d\theta$$

$$p(\mathbf{x} \mid e = 1) \propto \exp\left(\sum_i x_i b_i + x_+^2\right)$$





$$p(\mathbf{x}) = \int_R p(\mathbf{x} \mid \theta)\, f(\theta)\, d\theta$$

$$p(\mathbf{x} \mid \theta) = \frac{\exp\left(\sum_i x_i b_i + x_+\theta\right)}{\prod_i \exp\left(+[\theta + b_i]\right) + \exp\left(-[\theta + b_i]\right)}$$

$$p(\mathbf{x} \mid e = 1) = \frac{\exp\left(\sum_i x_i b_i\right)}{\prod_i \exp\left(+b_i\right) + \exp\left(-b_i\right)} \frac{\exp\left(x_+^2\right)}{\sup_{\mathbf{x}} \exp\left(x_+^2\right)}$$

$$p(\mathbf{x}, e) = \prod_i \frac{\exp(x_i b_i)}{\exp(+b_i) + \exp(-b_i)} \left(\frac{\exp(x_+^2)}{\sup_{\mathbf{x}} \exp(x_+^2)}\right)^e \left(1 - \frac{\exp(x_+^2)}{\sup_{\mathbf{x}} \exp(x_+^2)}\right)^{1-e}$$

common cause representation

- associations originate through an unobserved variable acting as a common cause with respect to the observed variables.

common effect representation

- associations arise from conditioning on a common effect of the observed variables.

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Project Details:	Joost Kruis MSc. | September 2015 - August 2019 | Funded by IOPS Graduate School PhD Grant 2015
Project Publication:	Kruis, J., Maris, G. (2016). *Three Representations of the Ising Model.* Manuscript submitted for publication.